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Divisor de voltaje y de corriente en inductores y capacitores con condiciones iniciales

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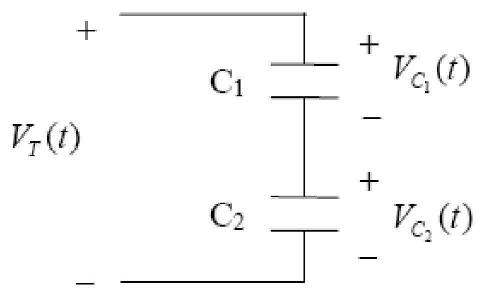
Divisor de Voltaje en capacitores:



$$V_T(t) - Vc_1(t) + Vc_2(t)$$

$$V_T(t) = \frac{1}{C_1}\int_{0^+}^{t} i_C(\tau)d\tau + Vc_1(0^+) + \frac{1}{C_2}\int_{0^+}^{t} i_C(\tau)d\tau + Vc_2(0^+)$$

$$V_T(t) - Vc_1(0^+) - Vc_2(0^+) = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)\frac{C_1}{C_1}\int_{0^+}^{t} i_C(\tau)d\tau$$

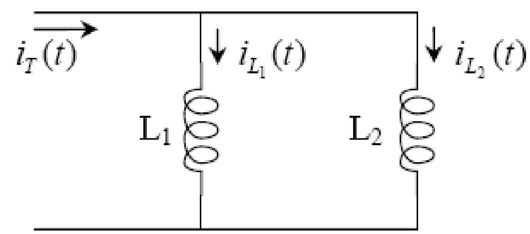
$$\left(V_T(t) - Vc_1(0^+) - Vc_2(0^+)\right)\frac{\frac{1}{C_1}}{\left(\frac{1}{C_1} + \frac{1}{C_2}\right)} = \frac{1}{C_1}\int_{0^+}^{t} i_C(\tau)d\tau$$

$$\boxed{Vc_1(t) = Vc_1(0^+) + \left(V_T(t) - V_T(0^+)\right)\frac{\frac{1}{C_1}}{\left(\frac{1}{C_1} + \frac{1}{C_2} + \ldots\right)}}$$

Si las condiciones iniciales son cero:

$$Vc_1(t) = V_T(t)\frac{\frac{1}{C_1}}{\left(\frac{1}{C_1} + \frac{1}{C_2} + \ldots\right)}$$

Divisor de Corriente en inductores:



$$i_T(t) = i_{L1}(t) + i_{L2}(t)$$

$$i_T(t) = \frac{1}{L_1}\int_{0^+}^{t} V_L(\tau)d\tau + i_{L1}(0^+) + \frac{1}{L_2}\int_{0^+}^{t} V_L(\tau)d\tau + i_{L2}(0^+)$$

$$i_T(t) - i_{L1}(0^+) - i_{L2}(0^+) = \left(\frac{1}{L_1} + \frac{1}{L_2}\right)\frac{L_1}{L_1}\int_{0^+}^{t} V_L(\tau)d\tau$$

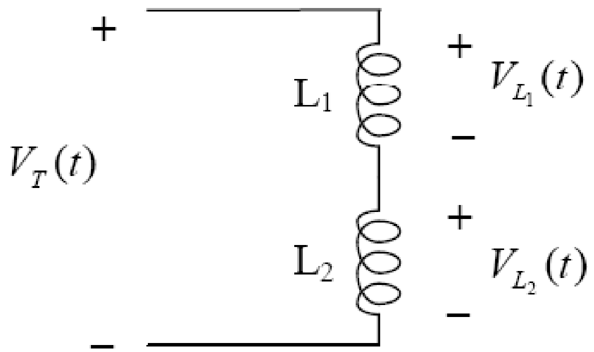
$$\left(i_T(t) - i_{L1}(0^+) - i_{L2}(0^+)\right)\frac{\frac{1}{L_1}}{\left(\frac{1}{L_1} + \frac{1}{L_2}\right)} = \frac{1}{L_1}\int_{0^+}^{t} V_L(\tau)d\tau$$

$$\boxed{i_{L1}(t) = i_{L1}(0^+) + \left(i_T(t) - i_T(0^+)\right)\frac{\frac{1}{L_1}}{\left(\frac{1}{L_1} + \frac{1}{L_2} + \ldots\right)}}$$

Si las condiciones iniciales son cero:

$$i_{L_1}(t) = i_T(t)\frac{\frac{1}{L_1}}{\left(\frac{1}{L_1} + \frac{1}{L_2} + \ldots\right)}$$

Divisor de voltaje en inductores:

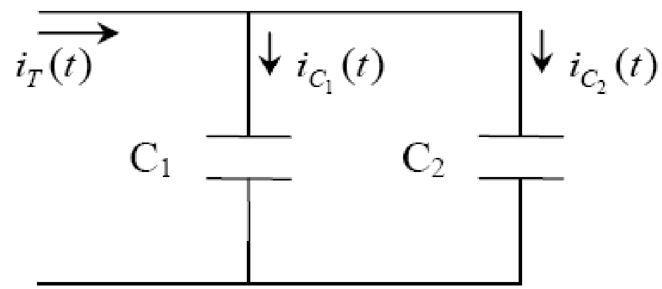


$$V_T(t) = V_{L1}(t) + V_{L2}(t)$$

$$V_T(t) = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} = (L_1 + L_2)\frac{1}{L_1} L_1 \frac{di}{dt}$$

$$\boxed{V_{L1}(t) = V_T(t) \frac{L_1}{L_1 + L_2 + \ldots}}$$

Divisor de corriente en capacitores:



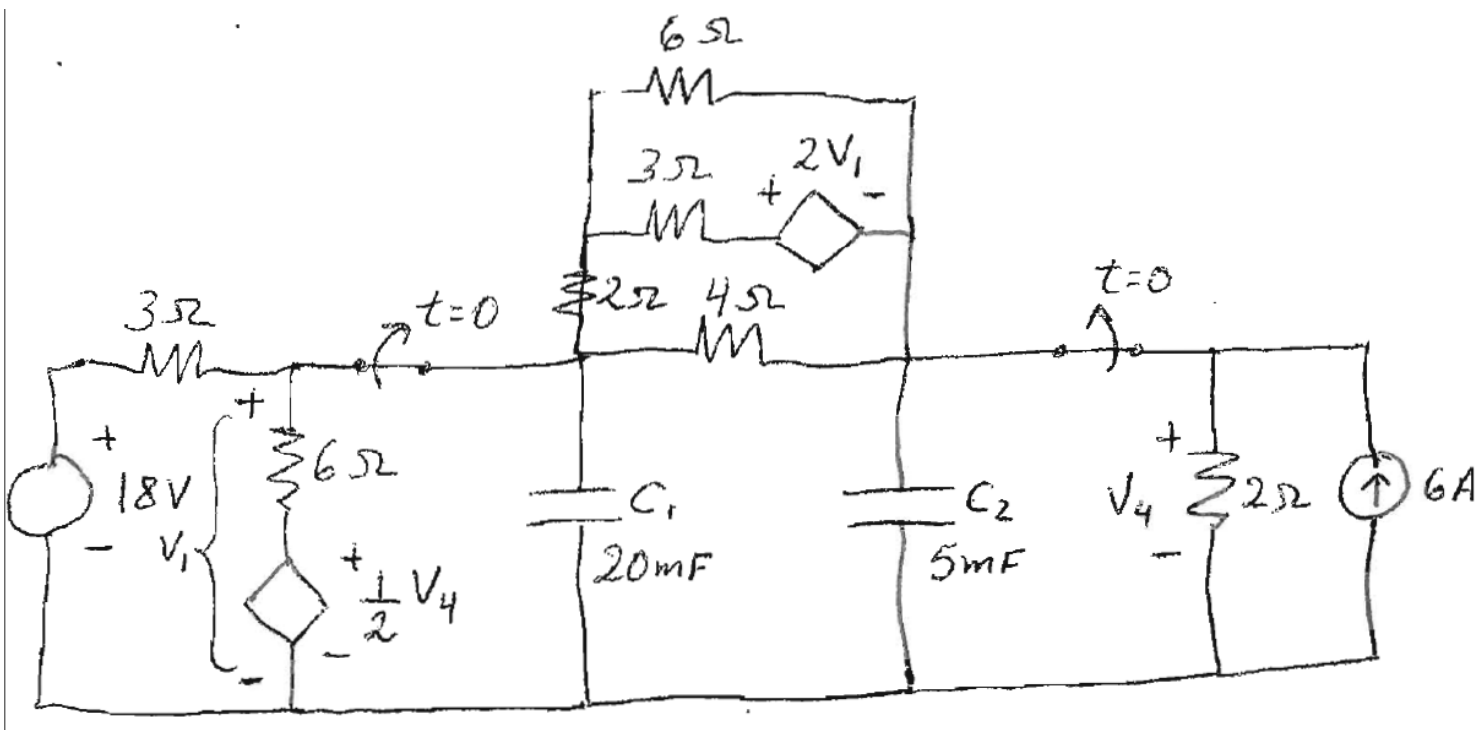
$$i_T(t) = i_{C1}(t) + i_{C2}(t)$$

$$i_T(t) = C_1 \frac{dV}{dt} + C_2 \frac{dV}{dt} = (C_1 + C_2)\frac{1}{C_1} C_1 \frac{dV}{dt}$$

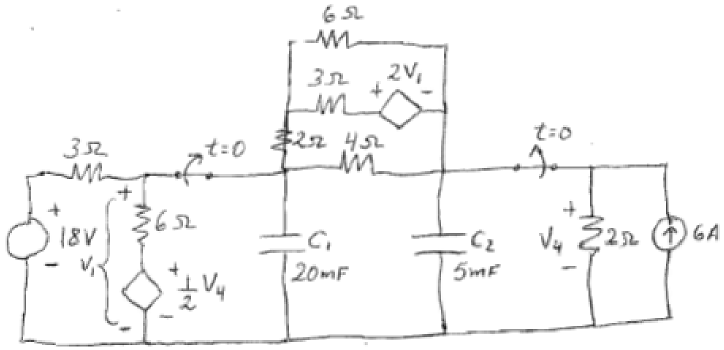
$$\boxed{i_{C1}(t) = i_T(t) \frac{C_1}{C_1 + C_2 + \ldots}}$$

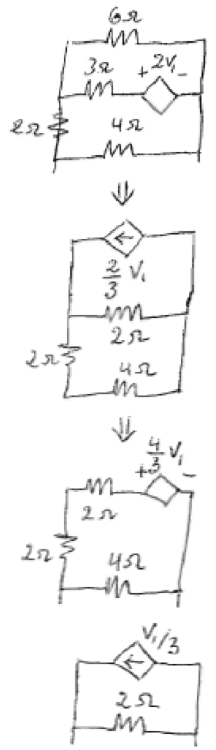
Ejercicio con 2 capacitores:

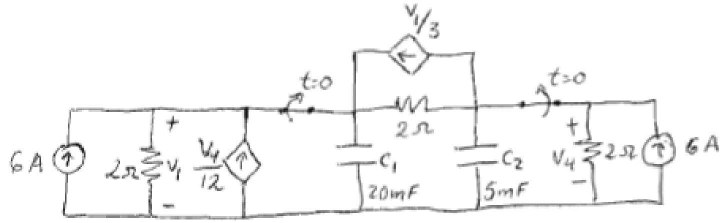
- Ver archivo Capacitores_en_serie2.pdf en la página web: http://webdelprofesor.ula.ve/ingenieria/ceballos



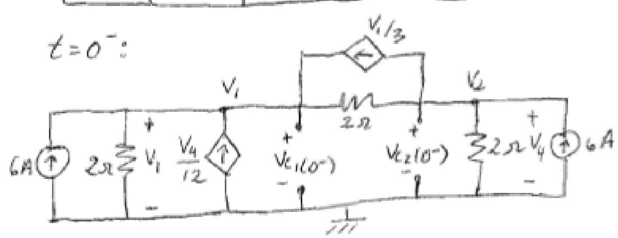
Ejercicio con 2 capacitores:







$t=0^-$:



MÉTODO DE VOLTAJES DE NODOS:

$$\left(\frac{1}{2}+\frac{1}{2}\right)V_1 - \frac{1}{2}V_2 = 6 + \frac{V_4}{12} + \frac{V_1}{3} \quad (V_4 \to V_2) \Rightarrow \frac{2}{3}V_1 - \frac{7}{12}V_2 = 6 \quad (1)$$

$$-\frac{1}{2}V_1 + \left(\frac{1}{2}+\frac{1}{2}\right)V_2 = 6 - \frac{V_1}{3} \Rightarrow -\frac{1}{6}V_1 + V_2 = 6 \quad (2)$$

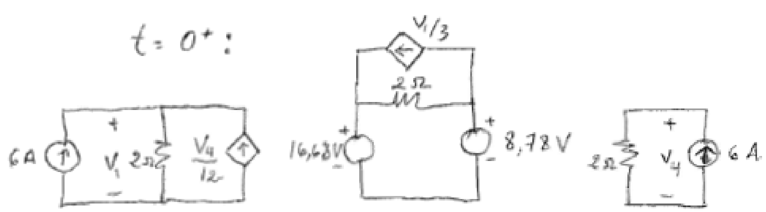
$\frac{1}{3} - \frac{1}{2} = \frac{2}{6} - \frac{3}{6}$

$V_1 = 16{,}68\ V$

$V_2 = 8{,}78\ V$

$V_{c_1}(0^-) = 16{,}68\ V \quad V_{c_2}(0^-) = 8{,}78\ V$

$t = 0^+$:

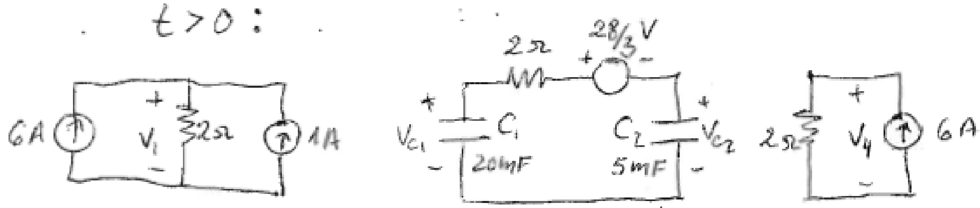


$V_{c_1}(0^+) = V_{c_1}(0^-) = 16{,}68V$

$V_{c_2}(0^+) = V_{c_2}(0^-) = 8{,}78V$

Ejercicio con 2 capacitores:

$t > 0$:

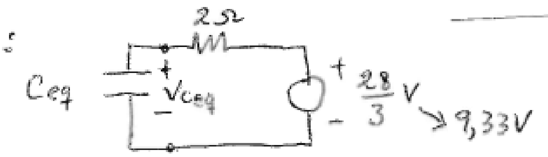


$V_4 = 12\,V$

$V_1 = 14\,V$

Equivalente de Thevenin:

$C_{eq} = \frac{20m \cdot 5m}{25m} = 4\,mF$



$$V_{Ceq}(t) = V_{Ceq}(\infty) + (V_{Ceq}(0^+) - V_{Ceq}(\infty))\, e^{-\frac{t}{RC}}$$

($V_{Ceq}(\infty) \to V_{TH}$; $V_{Ceq}(\infty) \to V_{TH}$; $R \to R_{TH}$; $C \to C_{eq}$)

$\tau = R_{TH} \cdot C_{eq} = 2 \cdot 4m = 8ms$

$t_s = 5\tau = 40ms$

$$V_T(t) = 9{,}33 + (7{,}9 - 9{,}33)\, e^{-\frac{t}{8m}}\ V$$

$$V_T(t) = 9{,}33 - 1{,}43\, e^{-\frac{t}{8m}}\ V$$

$V_{ceq}(0^+) = V_{c_1}(0^+) - V_{c_2}(0^+)$

$V_{ceq}(0^+) = 16{,}68 - 8{,}78$

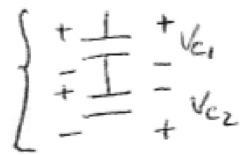
$V_{ceq}(0^+) = 7{,}9\ V$

DIVISOR DE TENSIÓN EN CAPACITORES:

V_1': $V_{C_1}(t) = V_{C_1}(0^+) + (V_{T(t)} - V_{T(0^+)})\, \frac{C_2}{C_1 + C_2}$

$$V_{C_1}(t) = 16{,}68 + (9{,}33 - 1{,}43\, e^{-\frac{t}{8m}} - 7{,}9)\, \frac{5m}{25m}$$

$$V_{C_1}(t) = 16{,}96 - 0{,}28\, e^{-\frac{t}{8m}}\ V$$



$V_{ceq} = V_1' + V_2' = V_{c_1} - V_{c_2}$

V_2': $-V_{C_2}(t) \Rightarrow V_2'(t) = V_2'(0^+) + (V_{T(t)} - V_{T(0^+)})\, \frac{C_1}{C_1 + C_2}$

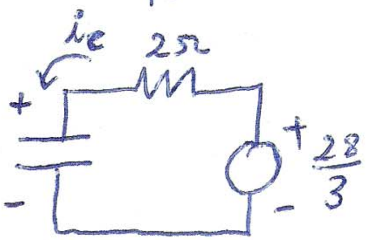
$$V_2'(t) = -8{,}78 + (9{,}33 - 1{,}43\, e^{-\frac{t}{8m}} - 7{,}9)\, \frac{20m}{25m}$$

$$V_2'(t) = -7{,}63 - 1{,}14\, e^{-\frac{t}{8m}}$$

$$V_{C_2}(t) = 7{,}63 + 1{,}14\, e^{-\frac{t}{8m}}$$

Otra forma, sin aplicar división de Tensión:

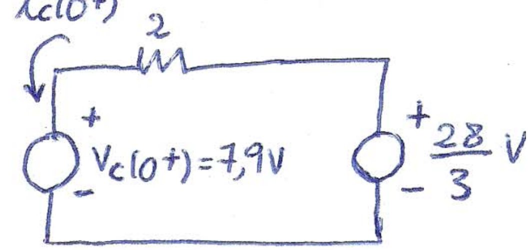
Hallar la corriente por el capacitor equivalente, que es la misma por los dos condensadores:



$$i_c(t) = i_c(\infty) + (i_c(0^+) - i_c(\infty))e^{-\frac{t}{\tau}}$$ (con $i_c(\infty) = 0$)

$$i_c(t) = 0{,}715\, e^{-\frac{t}{8m}}\ A$$

$t = 0^+$:



$$i_c(0^+) = \frac{9{,}33 - 7{,}9}{2}$$

$$i_c(0^+) = 0{,}715\ A$$

$$V_{c_1}(t) = V_{c_1}(0^+) + \frac{1}{C_1}\int_{0^+}^{t} 0{,}715\, e^{-\frac{\tau'}{8m}}\, d\tau'$$

$$= 16{,}68 + \frac{1}{20m}\cdot 0{,}715(-8m)\, e^{-\frac{\tau'}{8m}}\Big|_0^t$$

$$= 16{,}68 - (0{,}286\, e^{-\frac{t}{8m}} - 0{,}286) = 16{,}96 - 0{,}286\, e^{-\frac{t}{8m}}\ V$$

$$V_{c_2}(t) = V_{c_2}(0^+) + \frac{1}{C_2}\int_{0^+}^{t} \left(-0{,}715\, e^{-\frac{\tau'}{8m}}\right) d\tau'$$

$$= 8{,}78 + \frac{1}{5m}(-0{,}715)(-8m)\, e^{-\frac{\tau'}{8m}}\Big|_0^t$$

$$= 8{,}78 + (1{,}14\, e^{-\frac{t}{8m}} - 1{,}14) = 7{,}63 + 1{,}14\, e^{-\frac{t}{8m}}\ V$$

Ejercicio con 2 capacitores:

Código en Matlab para graficar:

```
t=0:0.001:0.1;%segundos
vc1=16.96-0.28*exp(-t/(8e-3));
vc2=7.63+1.14*exp(-t/(8e-3));
tms=t*1000;
figure
subplot(211)
plot(tms,vc1,'b')
title('Vc1')
xlabel('ms');ylabel('V')
subplot(212)
plot(tms,vc2,'r')
title('Vc2')
xlabel('ms');ylabel('V')
```